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Detecting Abraham's force of light by the Fresnel-Fizeau effect

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Abstract. The issue of the form that the energy tensor of the electromagnetic field should be given in matter is reconsidered, and the neat derivation of Abraham's tensor once provided by W. Gordon is recollected. In order to extend to the high frequency domain the experimental evidence gathered up to now in favour of Abraham's tensor, a method for detecting the Abraham's force supposedly exerted by light on a transparent, homogeneous medium is outlined. It avails of the Fresnel-Fizeau effect associated with the motion of matter that should be caused by the above mentioned force.

PACS. 03.50.-z Classical field theory – 04.20-q Classical general relativity – 42.65-k Nonlinear optics

1 Introduction

While a general consensus was eventually reached on the form to be given to the field equations of macroscopic electromagnetism, both in matter and *in vacuo*, the very form of the energy tensor of the electromagnetic field, hence the expression of the forces that the field should exert on matter, despite ingenuous efforts spread in a time span of more than one century, is still an open question. This state of affairs looks somewhat disappointing if one reminds that accounting for the electrodynamic forces was the *raison d'être* of the field conception itself. One is however forced to acknowledge that the revolution in physics happened at the beginning of this century has not helped in settling this problem, as it could have been expected at the outset. The radical changes in the theoretical approach to the whole set of questions concerning matter and radiation brought in by quantum mechanics and by quantum field theory, that so much have contributed to our present understanding, have cast a very dim light on the specific issue of the electrodynamic forces that prevail in matter. The "status of the electromagnetic energy-tensor", ridiculed by Synge in 1974 with a witty tale [1], is such that we still do not know for certain whether *e.g.* it was Minkowski [2] or Abraham [3] (just to quote the two best known proposals belonging to a huge theoretical literature) who came closer to a correct description of the electrodynamic forces in macroscopic matter.

Given the abundance of theoretical papers dealing with the problem, one could expect to meet with an equally rich experimental outflow, but, as well documented by the accurate report [4] published in 1979 by Brevik, that still constitutes an updated account, the latter production is rather sporadic and, more importantly, very few experiments survive a critical analysis as valuable pieces of evidence that discriminates between different theoretical options.

Among these few are the experiments proposed in 1955 by Marx and Györgyi [5], and performed in 1974 by Walker *et al.* [6], that are rated [4] as providing substantial support to the validity of the energy tensor proposed by Abraham. However, the experiments done by Walker *et al.* only deal with electromagnetic fields of very low frequency, while it would be theoretically important to ascertain whether light itself, being an electromagnetic phenomenon well described by Maxwell's equations[1], exerts Abraham's force on a transparent dielectric through which it happens to travel.

The present paper is meant to outline the theoretical underpinnings of a possible experiment. After recalling, in Section 2, the conceptually straightforward derivation [7] of Abraham's energy tensor given long ago by Gordon, in Section 3 it is shown how one can avail of the Fresnel-Fizeau effect for detecting the Abraham's force exerted by light on a transparent medium.

2 Gordon's derivation of Abraham's energy tensor

Gordon's far-sighted paper of 1923 produces for the first time the idea that electromagnetism in a linear, nondispersive medium that is homogeneous and isotropic in its rest frame can be reduced to electromagnetism in the vacuum of general relativity, as soon as one avails of a suitably defined "effective" metric, endowed *inter alia* with a remarkable physical property: its null geodesics, in the limit of geometrical optics, describe the rays along

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[1] At least as far as quantum features do not force us to jump to a different paradigm.

which light propagates in the medium. The same effective metric allows one to bring the choice of the Lagrangian for the electromagnetic field in the medium back to the case of the general relativistic vacuum, and to select uniquely, through the well established procedure [8] inaugurated by Hilbert, Abraham's tensor as the energy tensor for the electromagnetic field in the above mentioned matter. While Gordon's finding about the null geodesics was further considered and extended [9], the result about Abraham's tensor, to our knowledge [10], has been completely forgotten. We shall reproduce it here *in extenso* because, apart from the other virtues of its general relativistic formulation, Gordon's argument provides the shortest and most transparent way for showing the reader how and why such a thing as Abraham's energy tensor may be arrived at.

2.1 The field equations and the constitutive relations of electromagnetism

As stressed by Schrödinger [11], the unconnected space-time manifold suffices for writing Maxwell's equation in the naturally invariant form:

$$\mathbf{H}^{ik}_{\ ,k} = \mathbf{s}^i, \tag{1}$$

$$F_{[ik,m]} = 0, \tag{2}$$

where $\mathbf{H}^{ik}$ is a skew, contravariant tensor density that represents the electric displacement and the magnetic field, while F_{ik} is a covariant skew tensor that accounts for the electric field and for the magnetic induction in the well known way [12]. Of course, even if the four-current density $\mathbf{s}^i$ is prescribed *a priori*, these equations are not sufficient for determining, in a given co-ordinate system, both $\mathbf{H}^{ik}$ and F_{ik}. They fulfil the two identities:

$$\mathbf{H}^{ik}_{\ ,k,i} = 0, \tag{3}$$

which ensures the conservation of the electric four-current, and

$$\mathbf{e}^{ikmn} F_{[ik,m],n} = 0, \tag{4}$$

where $\mathbf{e}^{ikmn}$ is the totally antisymmetric tensor density of Ricci and Levi Civita. As the mere count of field components, equations and identities already suggests, Maxwell's equations need to be complemented by the so-called constitutive equations, which can take the form of a tensor equation that uniquely defines for instance $\mathbf{H}^{ik}$ in terms of F_{ik} and of the other fields that one may believe helpful in figuring out the electromagnetic medium. For a linear medium the constitutive equations can be written as [13]:

$$\mathbf{H}^{ik} = \frac{1}{2}\mathbf{X}^{ikmn} F_{mn}, \tag{5}$$

where the properties of the medium are specified by the four-index tensor density $\mathbf{X}^{ikmn}$. If the medium is the vacuum of general relativity, only the metric tensor g_{ik} in algebraic form will enter its definition:

$$\mathbf{X}^{ikmn} \equiv \sqrt{g}(g^{im}g^{kn} - g^{in}g^{km}), \tag{6}$$

where $g \equiv -\det(g_{ik})$, and the constitutive equation will read

$$\mathbf{H}^{ik} = \mathbf{F}^{ik} \equiv \sqrt{g} g^{im} g^{kn} F_{mn}. \tag{7}$$

2.2 The field equations and the energy tensor of electromagnetism for the vacuum of general relativity

Let us remind that in a manifold equipped with the pseudo Riemannian metric tensor g_{ik} a generic skew tensor F_{ik} can always be written as the sum of the curl of a potential φ and of the tensor dual to the curl of an "antipotential" ψ:

$$F_{ik} = \varphi_{k,i} - \varphi_{i,k} + e_{ik}^{\ \ mn}(\psi_{n,m} - \psi_{m,n}), \tag{8}$$

where the mixed tensor $e_{ik}^{\ \ mn}$ is obtained from $\mathbf{e}^{ikmn}$ with the usual procedure. If we start from the Lagrangian density

$$\mathbf{L} = \frac{1}{4}\mathbf{F}^{ik} F_{ik} - \mathbf{s}^i \varphi_i, \tag{9}$$

Hamilton's principle will yield *both* sets of Maxwell's equations:

$$\mathbf{F}^{ik}_{\ ,k} = \mathbf{s}^i, \tag{10}$$

$$F_{[ik,m]} = 0, \tag{11}$$

by subjecting φ *and* ψ to independent variations [14]. The Hamiltonian derivative of $\mathbf{L}$ with respect to the metric tensor will instead provide the energy tensor density of the electromagnetic field [8] that prevails in the gravitational vacuum:

$$\mathbf{T}_{ik} \equiv 2\frac{\delta \mathbf{L}}{\delta g^{ik}} = \mathbf{F}_i^{\ n} F_{kn} - \frac{1}{4} g_{ik} \mathbf{F}^{mn} F_{mn}. \tag{12}$$

This expression for the energy tensor holds provided that the homogeneous set of Maxwell's equations is satisfied.

2.3 The constitutive equations for a homogeneous, isotropic medium and Gordon's effective metric

The constitutive equations of such a medium were given by Minkowski for the special theory of relativity [2], and can be extended without change to the general theory. If u^i is the four-velocity of matter, by adopting Gordon's conventions for the metric g_{ik} we shall write $u^i u_i = -1$. Let us define the four-vectors:

$$F_i = F_{ik} u^k, \ H_i = H_{ik} u^k; \tag{13}$$

in general relativity an electromagnetic medium can be told homogeneous and isotropic if its constitutive equations can be written as

$$H_i = \epsilon F_i, \tag{14}$$

$$u_i F_{km} + u_k F_{mi} + u_m F_{ik} = \mu\big[u_i H_{km} + u_k H_{mi} + u_m H_{ik}\big], \qquad (15)$$

where the numbers ϵ and μ account for the dielectric constant and for the magnetic permeability of the medium. As shown by Gordon [7], these eight equations, that entail two identities, are equivalent to the six equations

$$\mu H^{ik} = F^{ik} + (\epsilon\mu - 1)(u^i F^k - u^k F^i) \qquad (16)$$

that provide the constitutive relation in standard form. Gordon noticed that the previous equation can also be written as

$$\mu H^{ik} = \big[g^{ir} - (\epsilon\mu - 1)u^i u^r\big]\big[g^{ks} - (\epsilon\mu - 1)u^k u^s\big]F_{rs}. \qquad (17)$$

The very form of the latter equation suggests to define the "effective metric tensor"

$$\gamma^{ik} = g^{ik} - (\epsilon\mu - 1)u^i u^k, \qquad (18)$$

that allows casting the constitutive equation into the form

$$\mu \mathbf{H}^{ik} = \sqrt{g}\gamma^{ir}\gamma^{ks}F_{rs}. \qquad (19)$$

The inverse of γ^{ik} is

$$\gamma_{ik} = g_{ik} + \big(1 - \frac{1}{\epsilon\mu}\big)u_i u_k; \qquad (20)$$

since $g \equiv -\det(g_{ik})$, one poses $\gamma \equiv -\det(\gamma_{ik})$, and finds [7]:

$$\gamma = \frac{g}{\epsilon\mu}. \qquad (21)$$

Therefore the constitutive equation eventually comes to read:

$$\mathbf{H}^{ik} = \sqrt{\frac{\epsilon}{\mu}}\sqrt{\gamma}\gamma^{ir}\gamma^{ks}F_{rs}, \qquad (22)$$

i.e., apart from a constant factor, coincides with the constitutive relation of a general relativistic "vacuum" defined by the effective metric tensor γ_{ik}.

2.4 The choice of the Lagrangian, and the energy tensor deriving from it

As put by Lanczos in his account [15] of Gordon's paper written for "Physikalische Berichte"[2]:

> "Durch Zurückführung aus das Vakuum hat man die Bequemlichkeit, unmittelbar das Prinzip der kleinsten Wirkung anwenden zu können, und erhält so den Abrahamschen elektromagnetischen Energie tensor des Feldes in ponderablen Körpern."

[2] Through the reduction to the vacuum one has the advantage to can immediately apply the least action principle, and obtains in this way Abraham's electromagnetic energy tensor of the field in ponderable bodies.

Given the analogy with the vacuum, the choice of the Lagrangian for the electromagnetic field in the medium is in fact immediate:

$$\mathbf{L}' = \frac{1}{4}\sqrt{\frac{\epsilon}{\mu}}\sqrt{\gamma}F^{(i)(k)}F_{ik} - \mathbf{s}^i\varphi_i, \qquad (23)$$

where, as before, one can write

$$F_{ik} = \varphi_{k,i} - \varphi_{i,k} + \frac{1}{\sqrt{\gamma}}\mathbf{e}_{(i)(k)}{}^{mn}(\psi_{n,m} - \psi_{m,n}); \qquad (24)$$

we have adopted the convention of enclosing within round brackets the indices that are either moved with γ_{ik} and γ^{ik}, or generated by performing the Hamiltonian derivative with respect to the just mentioned tensors. Independent variations of the action integral with respect to φ and ψ [14] will produce now Maxwell's equations (1) and (2). If the metric tensor of our pseudo-Riemannian space-time were γ_{ik}, we could obtain the energy tensor by executing the Hamiltonian derivative of the Lagrangian density $\mathbf{L}'$ with respect to the latter metric:

$$\delta\mathbf{L}' \equiv \frac{1}{2}\mathbf{T}'_{(i)(k)}\delta\gamma^{ik}, \qquad (25)$$

and we would get the mixed tensor density

$$\mathbf{T}'{}_{(i)}{}^{(k)} = F_{ir}\mathbf{H}^{kr} - \frac{1}{4}\delta_i{}^k F_{rs}\mathbf{H}^{rs}, \qquad (26)$$

which is just Minkowski energy tensor density in general relativistic form. But g_{ik}, not γ_{ik}, is the true metric that accounts for the structure of space-time and, through the Einstein tensor, defines its overall energy tensor. Therefore the partial contribution to that energy tensor coming from the electromagnetic field must be obtained by calculating the Hamiltonian derivative of $\mathbf{L}'$ with respect to g_{ik}. After some algebra one easily gets the electromagnetic energy tensor:

$$T_i{}^k = F_{ir}H^{kr} - \frac{1}{4}\delta_i{}^k F_{rs}H^{rs} - (\epsilon\mu - 1)\Omega_i u^k, \qquad (27)$$

where

$$\Omega^i = -\big(T_k{}^i u^k + u^i T_{mn}u^m u^n\big) \qquad (28)$$

is Minkowski's "Ruh-Strahl" [2]. Since $\Omega^i u_i \equiv 0$, substituting (27) into (28) yields:

$$\begin{aligned}\Omega^i &= F_m H^{im} - F_m H^m u^i \\ &= u_k F_m\big(H^{ik}u^m + H^{km}u^i + H^{mi}u^k\big)\end{aligned} \qquad (29)$$

and one eventually recognizes that T_{ik} is the general relativistic extension of Abraham's tensor [3] for a medium that is homogeneous and isotropic according to the definition given above. With Gordon's conventions, the four-force density exerted by the electromagnetic field on the medium is given by (minus) the covariant divergence of the energy tensor density $\mathbf{T}_i{}^k$:

$$\mathbf{f}_i = -\mathbf{T}_i{}^k{}_{;k}. \qquad (30)$$

3 Proposal for detecting the Abraham's force exerted by light within a transparent medium

We abandon now the general relativistic framework that has allowed the straightforward derivation of Abraham's tensor, and we assume henceforth that space-time is flat and looked at from a Minkowskian co-ordinate system:

$$g_{ik} = \eta_{ik} \equiv \mathrm{diag}(1,1,1,-1). \tag{31}$$

We assume further that, in the absence of applied electromagnetic fields, the homogeneous, isotropic medium happens to be at rest in that co-ordinate system, *i.e.*:

$$u^\lambda = 0,\ u^4 = 1; \tag{32}$$

the Greek indices run over the spatial co-ordinates. Let the macroscopic four-current s^i be vanishing in our medium; despite this fact the force density exerted by the electromagnetic field on the medium is in general not vanishing, as it would be if the energy tensor had the form postulated by Minkowski, and is given by:

$$\mathbf{f}_\lambda = -\frac{\epsilon\mu - 1}{\mu}\left[F_{4\alpha}F_\lambda{}^\alpha\right]_{,4},\ \mathbf{f}_4 = 0. \tag{33}$$

This is a quite remarkable outcome: as soon as in our coordinate system the electromagnetic field displays a dependence on time we shall expect that the field interact with the medium by exerting a force on it, despite the fact that the latter is homogeneous and devoid of macroscopic charges and currents. Due to this peculiarity, the attempt to build a phenomenological quantum electrodynamics by starting from Abraham's tensor [16,17] in order to evade some unpalatable consequences of the approach inaugurated by Jauch and Watson [18] fails to produce "photons", since the operators representing the energy and the momentum of the field do not have common eigenvectors.

Written in m.k.s. units, Abraham's ordinary force density takes the well known expression:

$$\mathbf{f} = \frac{n^2-1}{c^2}\frac{\partial}{\partial t}\left(\mathbf{E}\times\mathbf{H}\right), \tag{34}$$

where n is the refractive index of the medium, c is the speed of light *in vacuo*, and the three-dimensional notation is adopted.

3.1 Abraham's force exerted by a modulated light wave sets the medium in macroscopic motion

A light wave in our medium can be modeled after, say, the plane polarized electromagnetic wave with wave vector k and angular frequency ω that happens to propagate along the x axis of a Cartesian co-ordinate system; the nonvanishing components of this field can read:

$$E_y = E_0\cos(kx-\omega t),\ B_z = \sqrt{\epsilon_0\epsilon\mu_0\mu}E_0\cos(kx-\omega t) \tag{35}$$

in m.k.s. units. If Abraham's tensor holds for light, such a wave should exert a quite sizeable force density on the medium already with small light intensities, but a detection of its presence is hopeless, since its macroscopic average is vanishingly small over experimentally affordable length and time scales. However Brevik [4] has shown that, if the amplitude of the light wave is modulated at a low frequency, a mechanical effect due to Abraham's force may become detectable at a macroscopic scale. Imagine that a light wave fully modulated in amplitude can be described by the nonvanishing electromagnetic field components:

$$E_y = E_0\cos(k_1x-\omega_1t)\cos(k_2x-\omega_2t), \tag{36}$$

$$B_z = \sqrt{\epsilon_0\epsilon\mu_0\mu}E_0\cos(k_1x-\omega_1t)\cos(k_2x-\omega_2t), \tag{37}$$

with

$$\frac{\omega_1}{k_1} = \frac{\omega_2}{k_2} = \frac{c}{n}, \tag{38}$$

where ω_2 is the angular frequency of the unmodulated light, ω_1 is the angular frequency of modulation, and $\omega_1 \ll \omega_2$. We assume henceforth that the propagation occurs in a medium for which $\mu = 1$. The Abraham's ordinary force density (34) corresponding to the modulated field is directed along the x axis and has the value:

$$f = n\frac{n^2-1}{4c}\epsilon_0E_0^2\frac{\partial}{\partial t}\left\{[1+\cos 2(k_1x-\omega_1t)]\right. \\ \left.\times[1+\cos 2(k_2x-\omega_2t)]\right\}; \tag{39}$$

it contains the low frequency component

$$f_{low} = n\frac{n^2-1}{2c}\epsilon_0E_0^2\omega_1\sin 2(k_1x-\omega_1t) \tag{40}$$

which is potentially detectable [4] although, since its intensity is proportional to ω_1, one must be prepared to confront a severe experimental challenge.

We assume the length L of the path of light in the transparent medium to be much shorter than the wavelength $\lambda_1 = 2\pi/k_1$ of the modulating wave; therefore one can disregard the spatial dependence of f_{low} and consider the force density felt by the medium at a macroscopic level as depending only on time. We assume further that the transparent medium is allowed to move in some way in the direction of the force and that, under the effect of the latter, it will undergo as a whole standing oscillations with velocity $v = v_0\sin(2\omega_1t+\varphi)$; both the amplitude v_0 and the phase φ of the motion will depend on the details of the experimental set-up, that have been discussed elsewhere [19]. We shall disregard here the surface forces that the finiteness of the medium will necessarily entail, since their effect on the motion can be separately detected[3].

[3] If the electromagnetic field crossing the dielectric medium has the space-time dependence described by equations (36, 37), the experimental device can be so arranged that the low frequency component of the surface forces [4] turns out to be, with the required accuracy, in quadrature with respect to the low frequency component of Abraham's force.

3.2 The motion of the medium affects the propagation of a second, unmodulated light wave

Since v_0 is certainly quite small with respect to the velocity of light c, we can approximate the four-velocity of the medium as:

$$u^i \approx (\beta, 0, 0, 1), \tag{41}$$

where

$$\beta = \frac{v_0}{c}\sin(2\omega_1 t + \varphi) \approx -\frac{v_0}{c}\sin[2(k_1 x - \omega_1 t) - \varphi]; \tag{42}$$

the term $k_1 x$ has a negligible value along the path of light in the medium, supposed to occur between $x = 0$ and $x = L$; it has been inserted for ease of calculation. Imagine now that a second wave of unmodulated, linearly polarized light having, say, the same frequency ω_2 as the first wave travels through the medium in the same direction, and that, due to some contrivance, no overlapping of the two beams can occur. We shall assume that when the first beam is absent, and the medium is at rest, the electromagnetic field that models the second beam has the nonvanishing components:

$$F_{24} = E'_0\cos(k_2 x - \omega_2 t),\ F_{12} = \sqrt{\epsilon\mu}E'_0\cos(k_2 x - \omega_2 t); \tag{43}$$

we have recovered the more transparent four-dimensional, relativistic notation, and the associated units. When the medium is set in macroscopic motion at the angular frequency ω_1 by the Abraham's force exerted by the first beam, equations (43) will cease to provide a solution to Maxwell's equations since, in keeping with (16), the constitutive relation has changed, although very slightly. Due to the smallness of the change, a perturbative approach truncated at the first order in β will suffice. Let δF^{ik} represent the variation experienced by the unperturbed F^{ik}; the first-order change undergone by H^{ik} shall be given by:

$$\begin{aligned}\mu\delta H^{ik} = \delta F^{ik} + (\epsilon\mu - 1)[(\delta^i_4\delta F^{4k} - \delta^k_4\delta F^{4i})\\ +\beta(\delta^i_1 F^{4k} - \delta^k_1 F^{4i} + \delta^i_4 F^{k1} - \delta^k_4 F^{i1})].\end{aligned} \tag{44}$$

Therefore, while the first-order correction to the unperturbed set of Maxwell's equations (2) simply reads:

$$\delta F_{[ik,m]} = 0, \tag{45}$$

the correction to the other set $\mathbf{H}^{ik}_{\ ,k} = 0$ turns out to be

$$\frac{1}{\mu}\delta F^{\lambda\rho}_{\ \ ,\rho} + \epsilon\delta F^{\lambda 4}_{\ \ ,4} = \frac{\epsilon\mu - 1}{\mu}\left[(\beta F^{4\lambda})_{,1} - (\beta F^{1\lambda})_{,4}\right] \tag{46}$$

for $i = \lambda$, and

$$\epsilon\delta F^{4\rho}_{\ \ ,\rho} = 0 \tag{47}$$

for $i = 4$. The first-order correction to the unperturbed Maxwell's equations hence displays with respect to δF^{ik} the very form that the unperturbed equations exhibit with F^{ik}, if one excepts the component with $\lambda = 2$ of equation (46), that can be written as:

$$\delta F^{21}_{\ \ ,1} + \epsilon\mu\delta F^{24}_{\ \ ,4} = 2(\epsilon\mu - 1)(\beta F^{42})_{,1} \tag{48}$$

since with our fields $(\beta F^{42})_{,1} = -(\beta F^{12})_{,4}$. Therefore only the corrections δF^{21} and δF^{42} to the unperturbed components on the field F^{ik} shall be nonvanishing. A physically appropriate particular solution to equations (48) and (45) can be obtained as follows. Let us set

$$\zeta = (2k_1 + k_2)x - (2\omega_1 + \omega_2)t - \varphi, \tag{49}$$

$$\eta = (2k_1 - k_2)x - (2\omega_1 - \omega_2)t - \varphi, \tag{50}$$

and define the nonvanishing component δA_2 of the first-order correction to the four-potential A_i as:

$$\delta A_2 = Cx[\sin\zeta + \sin\eta], \tag{51}$$

where C is a constant to be determined. When δF_{ik} is defined as the curl of δA_i equation (45) is fulfilled, and the resulting components of the first-order correction:

$$\begin{aligned}\delta F_{12} = C[\sin\zeta + \sin\eta\\ +(2k_1 + k_2)x\cos\zeta + (2k_1 - k_2)x\cos\eta],\end{aligned} \tag{52}$$

$$\delta F_{42} = -\frac{C}{\sqrt{\epsilon\mu}}\ [(2k_1 + k_2)x\cos\zeta + (2k_1 - k_2)x\cos\eta], \tag{53}$$

satisfy equation (48), provided that

$$C = \frac{1}{2}(\epsilon\mu - 1)\frac{v_0}{c}E'_0. \tag{54}$$

We gather from equations (52) and (53) that, due to the Fresnel-Fizeau effect [12] a monocromatic, plane polarized wave entering the medium at $x = 0$ gets modulated at the frequency of the motion caused by the Abraham's force; when x is so large that $|k_2 x| \gg 1$ the modulated part of the field can be approximately written as

$$\delta F_{12} \approx 4\pi C\frac{x}{\lambda_2}\sin(2\omega_1 t + \varphi)\sin(k_2 x - \omega_2 t), \tag{55}$$

$$\delta F_{24} \approx 4\pi\frac{C}{\sqrt{\epsilon\mu}}\frac{x}{\lambda_2}\sin(2\omega_1 t + \varphi)\sin(k_2 x - \omega_2 t), \tag{56}$$

where λ_2 is the wavelength of light in the considered optical medium. As the light of the second wave propagates through the latter a modulated component shows up, in quadrature with respect to the unmodulated field (43) and, according to the perturbative result, its amplitude grows up linearly with the distance from the origin. Therefore, provided that L is large enough, when compared to the wavelength of light, and that the intensity of the first beam is strong enough to impress to the medium an adequate speed in the back and forth motion , one can hope to prove that light indeed exerts Abraham's force on a transparent medium by detecting, with sensitive techniques, the change operated by the Fresnel-Fizeau effect on the second, originally unmodulated beam.

3.3 A possible experimental setup

It is not our intention to deprive the experimentalists from the demiurgic pleasure of figuring out how the theoretical exertions of the previous two subsections can be translated into an actual experimental device. However, already in his report, written in 1979, Brevik [4] suggested availing of a long glass fibre wound on the cylindrical drum of a torsion pendulum, and that the modulation frequency of the light sent into the fiber should be equal to one half the resonance frequency of the latter, in order to enhance as much as possible the amplitude of the motion supposedly caused by Abraham's force.

The great progress achieved since then in understanding the physical properties of the optical fibres [20] has allowed for great improvements in the art of manufacturing fibres and related optical devices with the desired physical properties. One is therefore tempted to avail of this highly developed branch of technology for enlarging the scant evidence gathered up to now about the force that, according to Abraham, electromagnetic fields should exert on the dielectric media through which they happen to propagate. To comply with the proposal of the present paper one should only add to the first fibre contemplated by Brevik a second one, in which a monochromatic, unmodulated beam with the angular frequency ω_2 is injected. An order of magnitude estimate of the modulation induced in the second beam by the Fresnel-Fizeau effect when an experimental device of this sort is operated within the presently achievable limits is provided in reference [19]. Since the modulated component of the second beam is in quadrature with respect to the unmodulated part, an interferometric method is suggested for the extraction of the low frequency signal that should be eventually detected[4].

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[4] The order of magnitude estimate of reference [19] shows that a signal detectable with a present day digital "lock in" should be obtained, if the torsion pendulum were built by winding on a cylinder of radius $R = 10$ cm two commercially available, single mode fibres, each one with a length $L = 1$ km. If Abraham's force is indeed exerted by light, a power of ~ 60 mW for the modulated light injected in the first fibre should be already sufficient for producing at the end of the second fibre an induced modulation such that the power ratio of the modulated component with respect to the unmodulated background would be $\sim 2.0 \times 10^{-10}$. The interested reader may find further details and suggestions in the "Festschrift" contribution mentioned above.